Electron-positron anomalous magnetic moments comparison at storage ring
(experiment 1987 & perspective)

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topics

- *spin in rings*
- *radiative polarization of $e^{\pm}$*
- *RF spin flip*
- *resonance dępolarization `*
- *spin tune spread*
- *experiment 1987*
- *systematics*
- *perspective*

Spin in accelerator

1959

BMT (Bargman, Michel, Telegdi) equation

$$\dot{\vec{S}} = \frac{d\vec{S}}{dt} = \left[\vec{\Omega} \times \vec{S}\right];$$

$\vec{\Omega}$ - spin precession frequency

$$-\vec{\Omega} = \left(1 + \gamma \frac{q'}{q_0}\right)\frac{q_0}{\gamma}\vec{B} + \frac{q}{\gamma}\vec{B}_{\parallel} + \left(\frac{q_0}{\gamma+1} + q'\right)\left[\vec{E} \times \vec{V}\right]_{\perp}$$

$$-\vec{\omega}_L = \frac{q_0}{\gamma}\vec{B}_{\perp} + \frac{\gamma q_0}{\gamma^2 - 1}\left[\vec{E} \times \vec{V}\right]$$

$q_0 = \frac{e}{m}$ *-normal and* q' *anomalous parts of gyromagnetic ratio*

$$\frac{\Omega}{\omega_L} = \nu \rightarrow spin\ tune$$

$$\nu_0 = \gamma \frac{q'}{q_0} = \gamma \cdot \mathrm{a}$$

$$\mathrm{a} = \frac{q'}{q_0} = G = \frac{g - 2}{2} \quad - magnetic\ anomaly$$

$$E = B_{\parallel} = 0; \qquad \nu = \nu_0; \qquad \nu_0 = 1 \quad (E = 440.649\ MeV)$$

Spin in accelerator

1959

Polarized proton acceleration at SATURN.

Froissart-Stora: spin resonance crossing

$$S(\infty) = S_0 \cdot \left(2 \cdot e^{-\psi} - 1\right)$$

$$\psi = \frac{\pi \cdot w_k^2}{2\dot{\delta}}$$

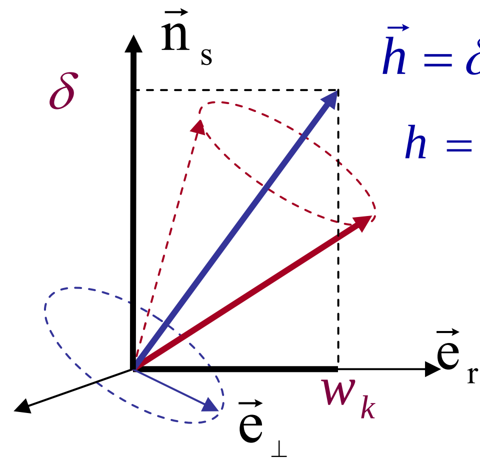
w_k **-resonance strength;**

$\delta = \nu - \nu_k$ **- tuning**

$\dot{\delta}$ **- tuning rate**

spin phase advance in resonance zone $\delta \approx w_k$

Single resonance model



$$\vec{h} = \delta \cdot \vec{n}_s + w_k \vec{e}_r$$

$$h = \sqrt{\delta'^2 + w_k^2}$$

$$\vec{n} = \frac{\vec{h}}{h}$$

Fast crossing: $(\psi \ll 1)$

$\dot{\delta}\uparrow \qquad w_k \downarrow$

$$S(\infty) \approx S_0; \qquad \delta S \approx S_0 \psi$$

Adiabatic crossing: $(\psi \gg 1)$

$\dot{\delta} \downarrow \qquad w_k \uparrow$

$$S(\infty) \approx -S_0; \quad \delta S \approx 2 S_0 e^{-\psi}$$

RF flipper! (VEPP-2M 1980)

Radiative polarization

1964

A.Sokolov
I.Ternov

$$P^{\uparrow\downarrow} = P_{cl}\left(\frac{\hbar\omega}{E}\right)^2\left(1 \pm \frac{35\sqrt{3}}{64}\right) \Rightarrow \zeta = \frac{8}{5\sqrt{3}}\left(1 - e^{-t/\tau}\right)$$

$$\tau_{ST} = \left(q_0^5\gamma^2 B^3\right)^{-1} \cong \frac{\left(R/\rho\right)}{\left[B(T)\right]^3\left[E(GeV)\right]^2}(hour)$$

1973

Ya.Derbenev
A.Kondratenko

$$\zeta_{\max} = \frac{\alpha_-}{\alpha_+}; \qquad \alpha_- = \left\langle \left|\vec{B}\right|^3 \vec{b}\cdot\left(\vec{n}_0 - \vec{d}\right)\right\rangle_\theta \; ; \qquad \vec{b} = \frac{\vec{B}}{B}$$

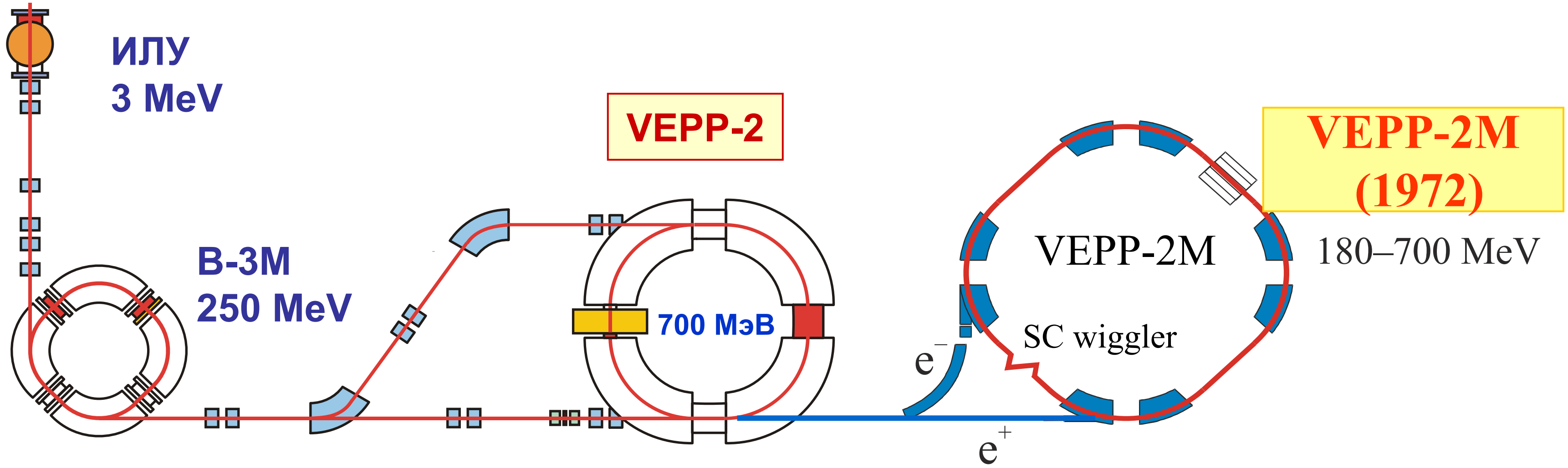
$$\alpha_+ = \tau_p^{-1} = \frac{5\sqrt{3}}{8} q_0^5\gamma^2 \left\langle \left|\vec{B}\right|^3 \left[1 - \frac{2}{9}\left(\vec{n}_0\cdot\vec{V}\right) + \frac{11}{18}\vec{d}^2\right]\right\rangle_\theta$$

$$\vec{d}(\theta) = \gamma\frac{\partial\vec{n}}{\partial\gamma}$$

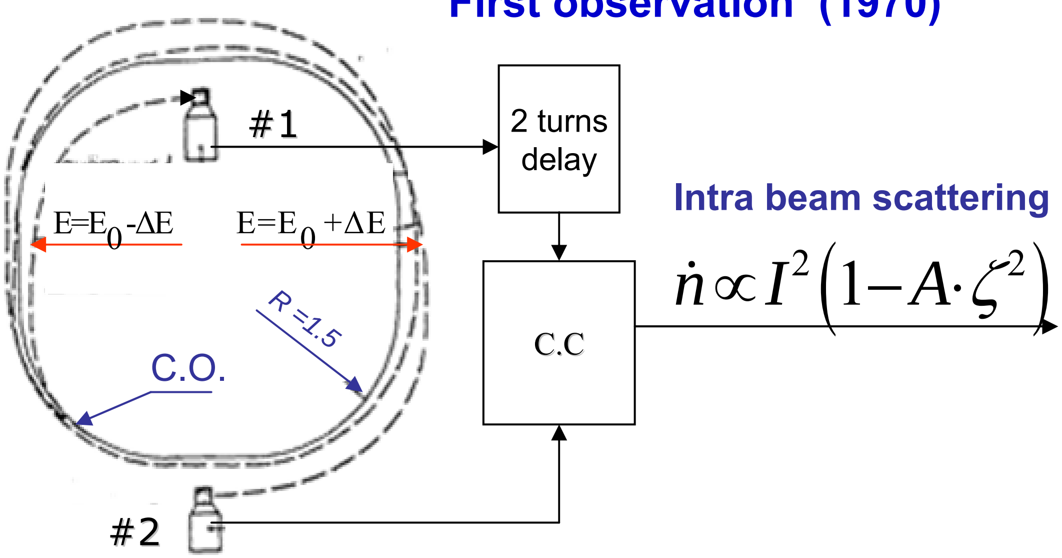
spin-orbit coupling vector

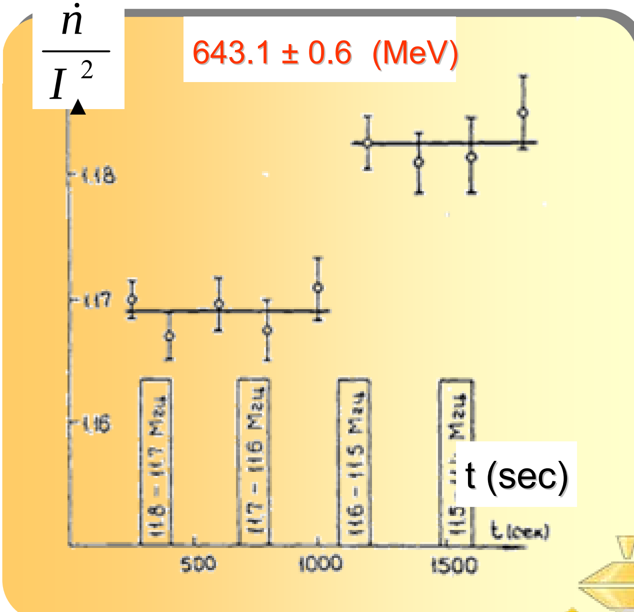
$$d \sim \gamma^2$$

Radiative polarization



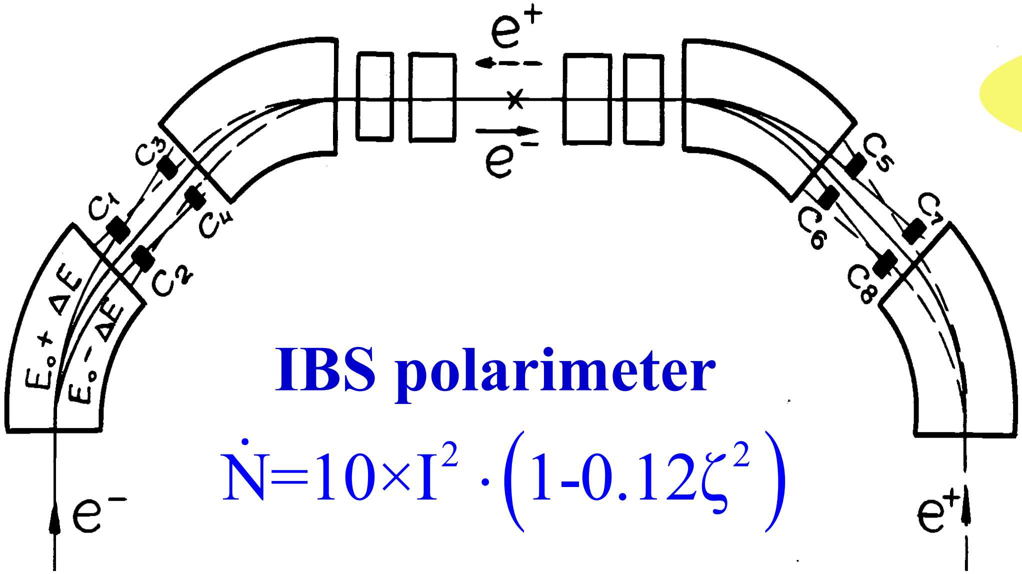
First observation (1970)





(Q_x=0.67)

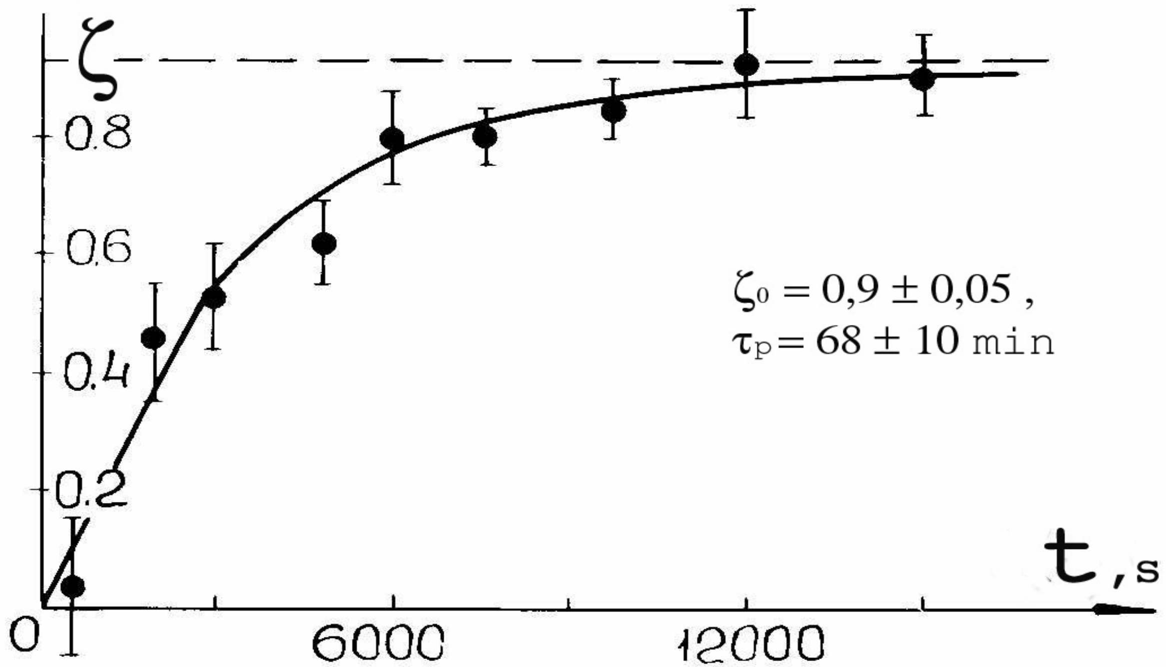
VEPP-2M (1974-1975)



IBS polarimeter

$$\dot{N}=10\times I^2\cdot\left(1-0.12\zeta^2\right)$$

E=670 MeV

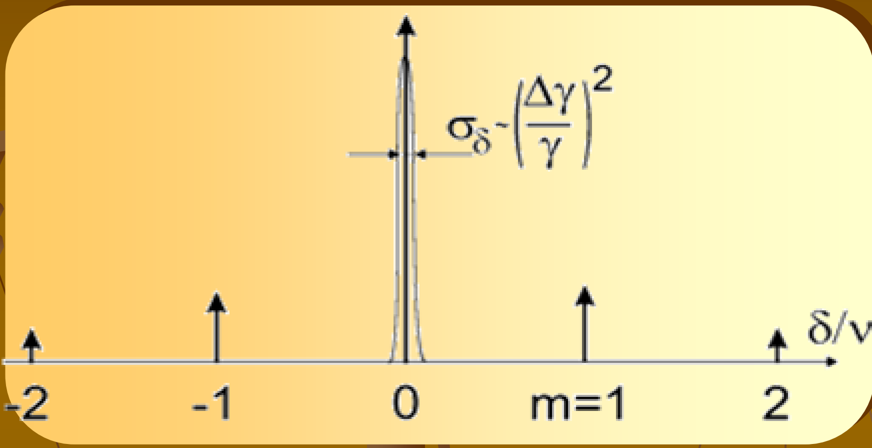


Absolute particle energy calibration

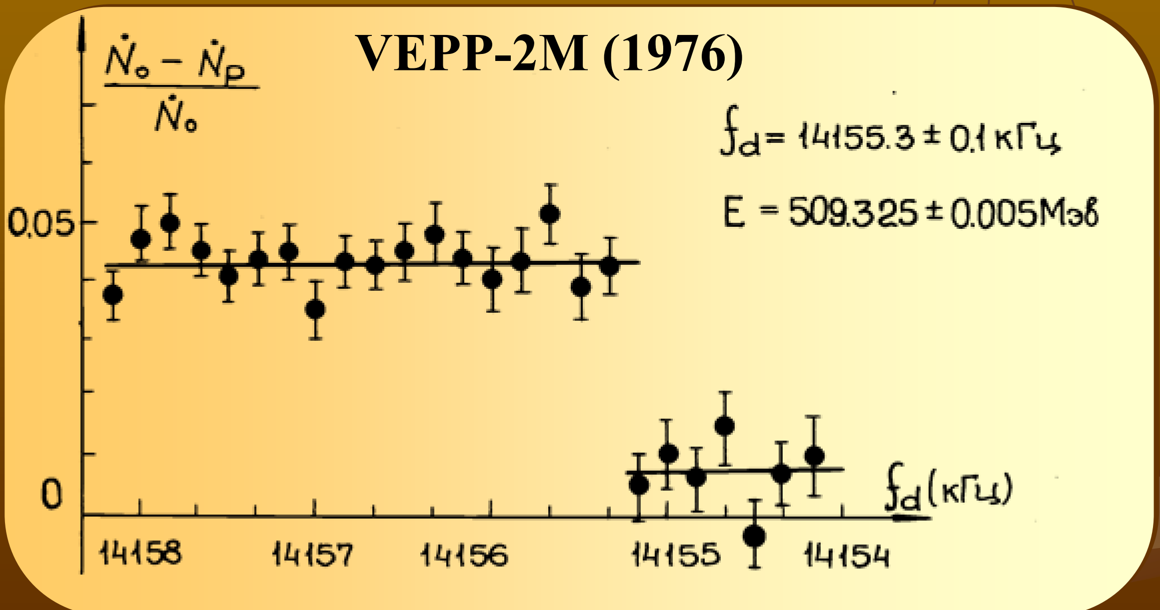
$$\nu = \frac{\Omega}{\omega_0} = \gamma \frac{q'}{q_0}$$ -spin tune; $\delta\left(\frac{q'}{q_0}\right) \approx 1\cdot 10^{-8}$; $\delta(m_e) \approx 3\cdot 10^{-7}$; $\frac{\Delta E}{E_0} \approx 10^{-3}$;

Synchrotron oscillations

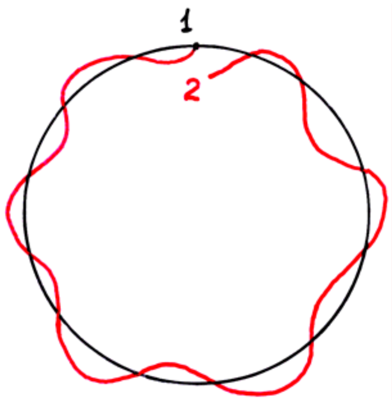
$$E = E_0\left(1 + \frac{\Delta E}{E_0}\cos(\nu_\gamma \theta)\right); \qquad \nu_\gamma \approx 10^{-2}$$



spin tune spread?



Spin tune spread (dynamics)



1. Delay

$$\Delta l \propto p_{\perp}^2 = c_x^2 \left(|f_x|'^2 + 1 \big/ |f_x|^2 \right)$$

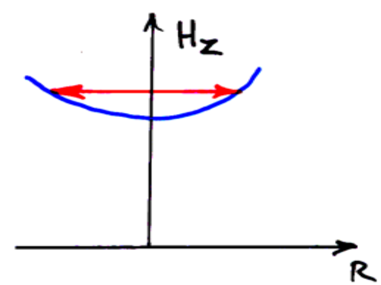
c_x - amplitude of oscillation,

f_x - Floquet function, $x = c_x f_x + c.c.$

Energy shift due to synchronism with RF $\delta\nu \approx 10^{-5}\omega_0$

2. magnetic field nonlinearities

$$E \propto \int_0^{2\pi} H_z d\theta; \qquad \Delta E \propto \left\langle \frac{\partial^2 H_z}{\partial x^2} x^2 \right\rangle.$$

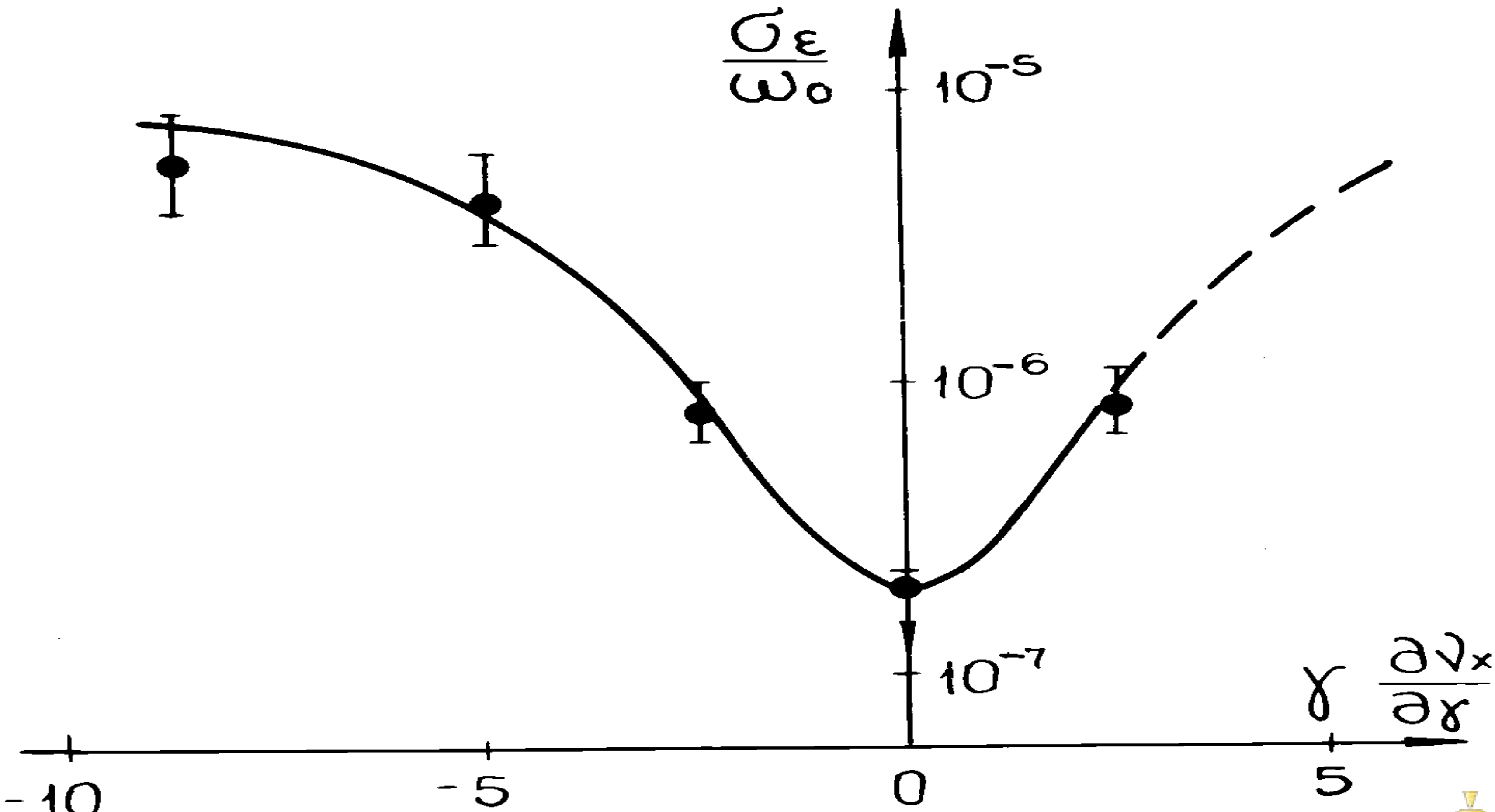


$$\langle \delta\nu \rangle = \sigma_\nu \approx \frac{\nu_0}{\alpha} c_x^2 \left\langle n_1 |f_x|^2 \psi_x - \left(|f_x|^2 + 1 \big/ |f_x|^2 \right) \right\rangle = \frac{\nu_0}{\alpha} c_x^2 \cdot \gamma \left(\frac{d\nu_x}{d\gamma} \right) \sim \gamma^3$$

$$n_1 = \frac{R_0^2}{B_0} \frac{d^2 B_z}{dx^2} \qquad \gamma \frac{d\nu_x}{d\gamma} \text{ - chromaticity;}$$

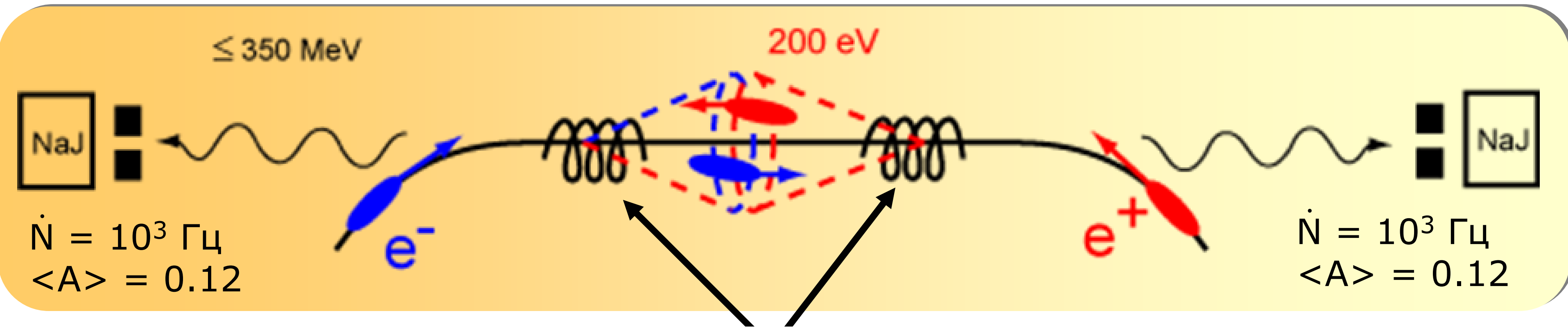
Control of spin tune spread by chromaticity

VEPP-2M: $\sigma_\nu \Rightarrow 10^{-7}\omega_0$

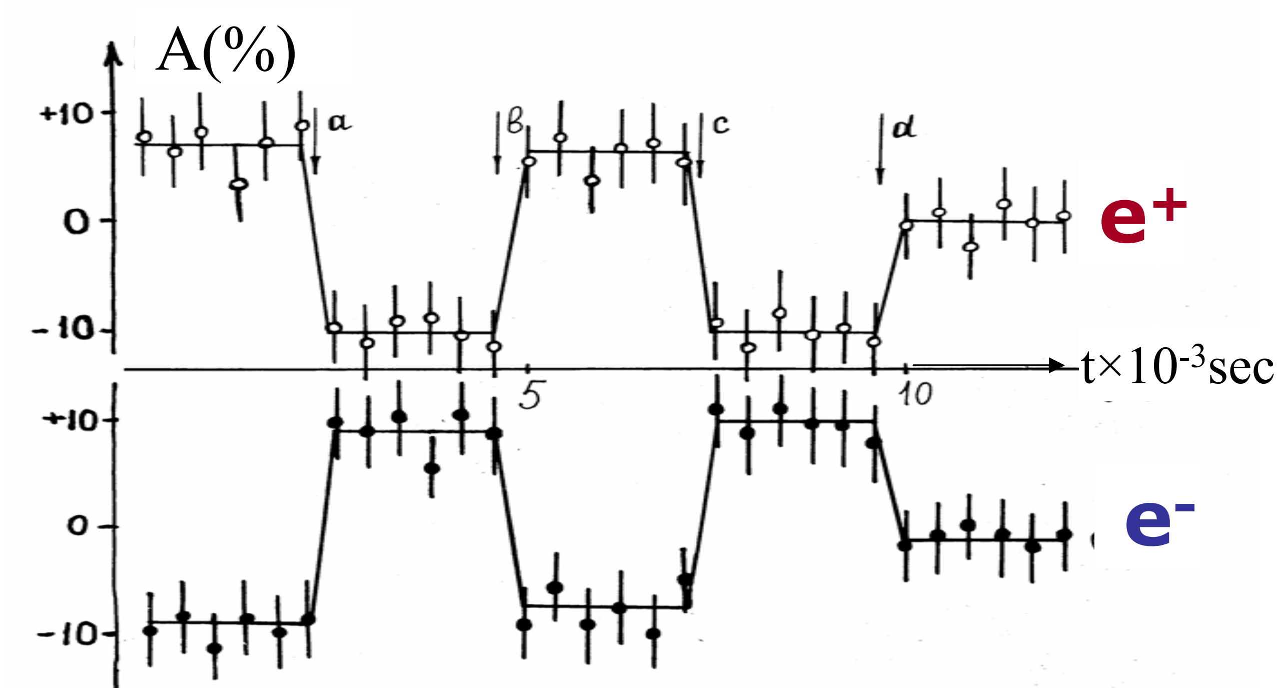


$\frac{\sigma_\varepsilon}{\omega_0}$
10^{-5}
10^{-6}
10^{-7}
$\gamma \frac{\partial \nu_x}{\partial \gamma}$
-10
-5
0
5

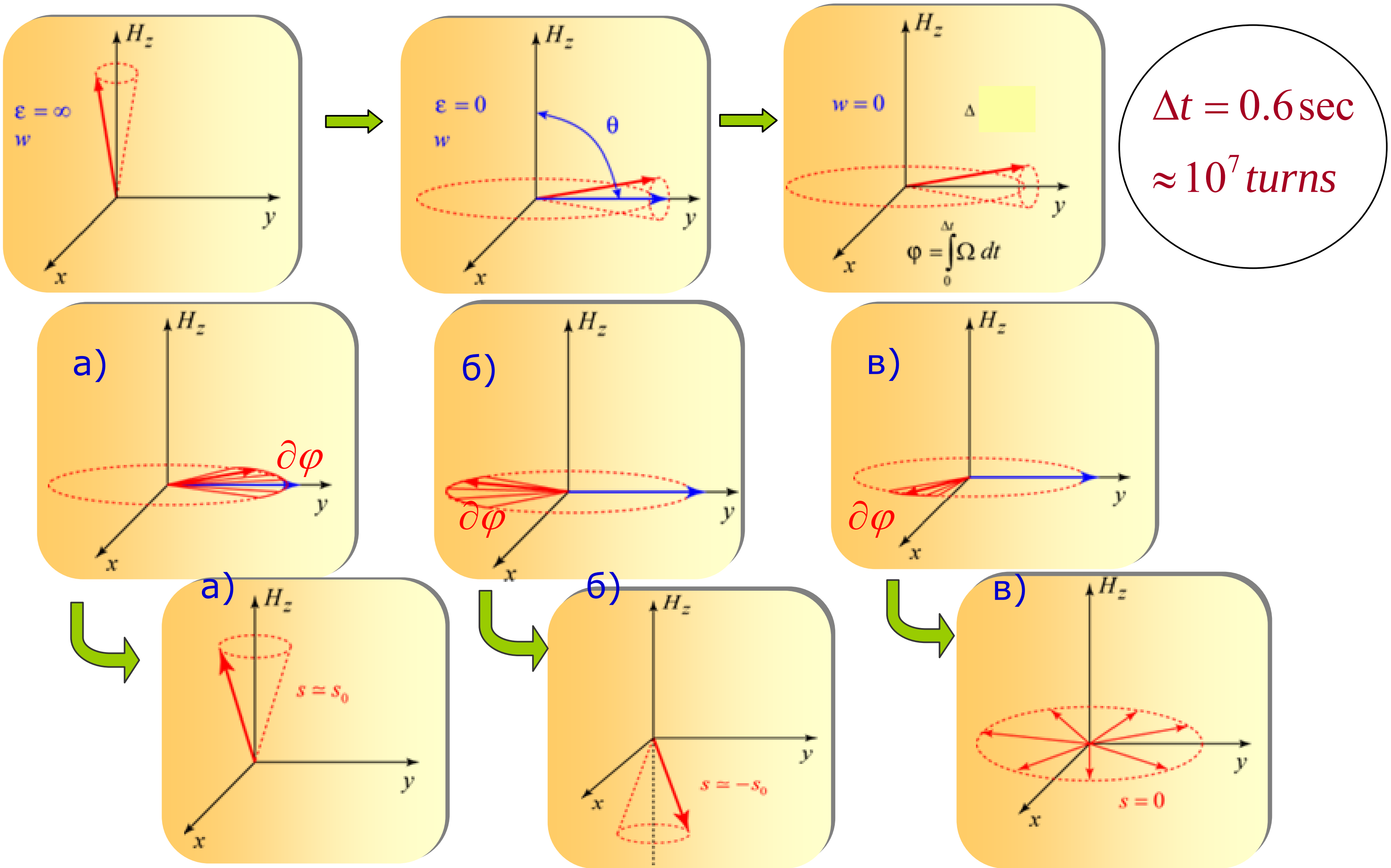
Compton polarimeter with helical magnets at VEPP-2M (E=670 MeV)



helical undulators (λ=2.4 cm; B=0.5 T)



e⁺e⁻ spin gymnastics

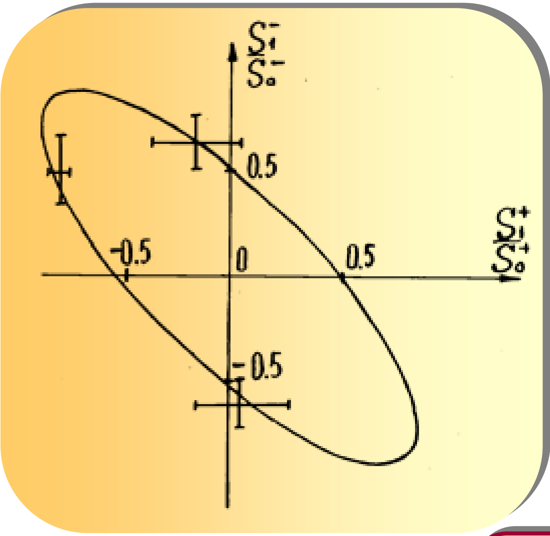


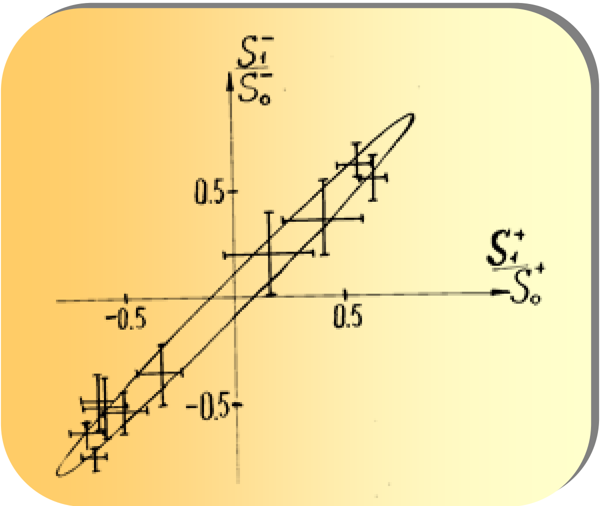
$$S_1^{\pm} = S_0^{\pm}(\cos^2\theta + \sin^2\theta \cdot \cos\varphi^{\pm} \cdot e^{-\frac{(\partial\varphi)^2}{2}})$$

e^+e^- anomalous magnetic moments comparison

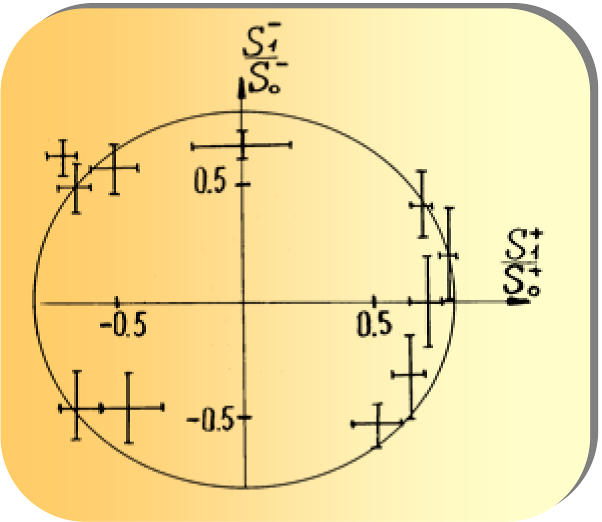
$\Delta\varphi = \varphi^+ - \varphi^- = (5 \pm 6)^0$

30 B





20 B



VEPP-2M (1987)

$$\frac{\gamma^- a^- - \gamma^+ a^+}{\bar{\gamma}} < 1 \cdot 10^{-8}$$

< 1987

$$a^- - a^+ = (2.2 \pm 6.4) \times 10^{-8}$$

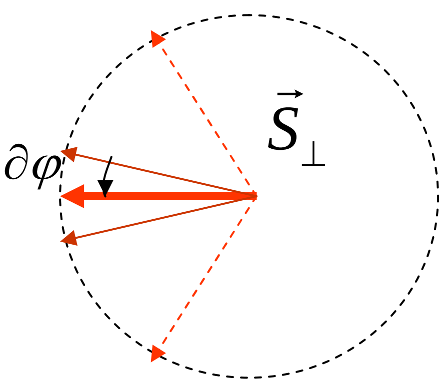
now

$$a^- - a^+ = (0.05 \pm 0.2) \times 10^{-8}$$

$$\Delta(a^-) = 0.024 \times 10^{-8}$$

Systematics I

"top view" in the frame rotating with frequency $\nu_0 \cdot f_0$



$$\vec{S}_\perp = S_\perp e^{i\cdot\chi\sin(\nu_\gamma\theta)} = S_\perp \sum_m J_m(\chi) e^{i\cdot m\nu_\gamma\theta}; \qquad \chi = \frac{\nu_0}{\nu_\gamma}\cdot\frac{\Delta\gamma}{\gamma} \approx 0.1$$

Average over the distribution

$$\frac{1}{\sigma_E} e^{-\frac{(\Delta E)^2}{2\sigma_E^2}} \Delta E \cdot d(\Delta E)\frac{d\varphi}{2\pi}$$

$$\left\langle \vec{\zeta} \right\rangle = \left\langle \frac{\vec{S}_\perp}{S_\perp} \right\rangle = \frac{1}{\sigma_E^2}\int J_0(\chi)\, e^{-\frac{(\Delta E)^2}{2\sigma_E^2}} \Delta E \cdot d(\Delta E) = e^{-\frac{1}{2}\chi^2}$$

Depolarization: *spin tune spread + quantum emission*

$$(\partial\varphi)^2 \approx (\delta\nu)^2 \tau_0 t; \Rightarrow \tau_d \approx \frac{1}{(\delta\nu)^2\cdot\tau_0} \sim \gamma^{-3}; \quad (\tau_d = 5\sec)$$

$\tau_0 \sim \gamma^{-3}$ -radiative damping time

$(\delta\nu) \sim \gamma^3$ -spin tune spread

Systematics II

$$\vec{\Omega} = \frac{q_0}{\gamma}\left\{(1+\gamma \mathrm{a})\vec{B}_{\perp} + (1+\mathrm{a})\vec{B}_{\parallel} + \left(\frac{\gamma}{\gamma+1} + \gamma \mathrm{a}\right)\left[\vec{E}\times\vec{V}\right]\right\}$$

$$\frac{\Omega^{+}-\Omega^{-}}{\Omega} \approx \frac{2\oint E_{\perp}dl}{\oint B_{\perp}dl} = \frac{V_{acc}(\phi=\phi_0)\cdot\alpha}{\pi 300 B\rho} = \frac{V_{acc}(\phi=\phi_0)\cdot\alpha}{\pi \mathrm{E}(\mathrm{MeV})}$$

$$V_{acc}(\phi=\phi_0) = \frac{90\cdot \mathrm{E}^4(\mathrm{MeV})}{\rho(m)}; \qquad \alpha = \measuredangle \vec{E}\,\&\,\vec{V}$$

VEPP-2M: E=670 MeV; $V_{acc}(\phi=\phi_0) = 15\ kV;\ \ \alpha < 10^{-3}$

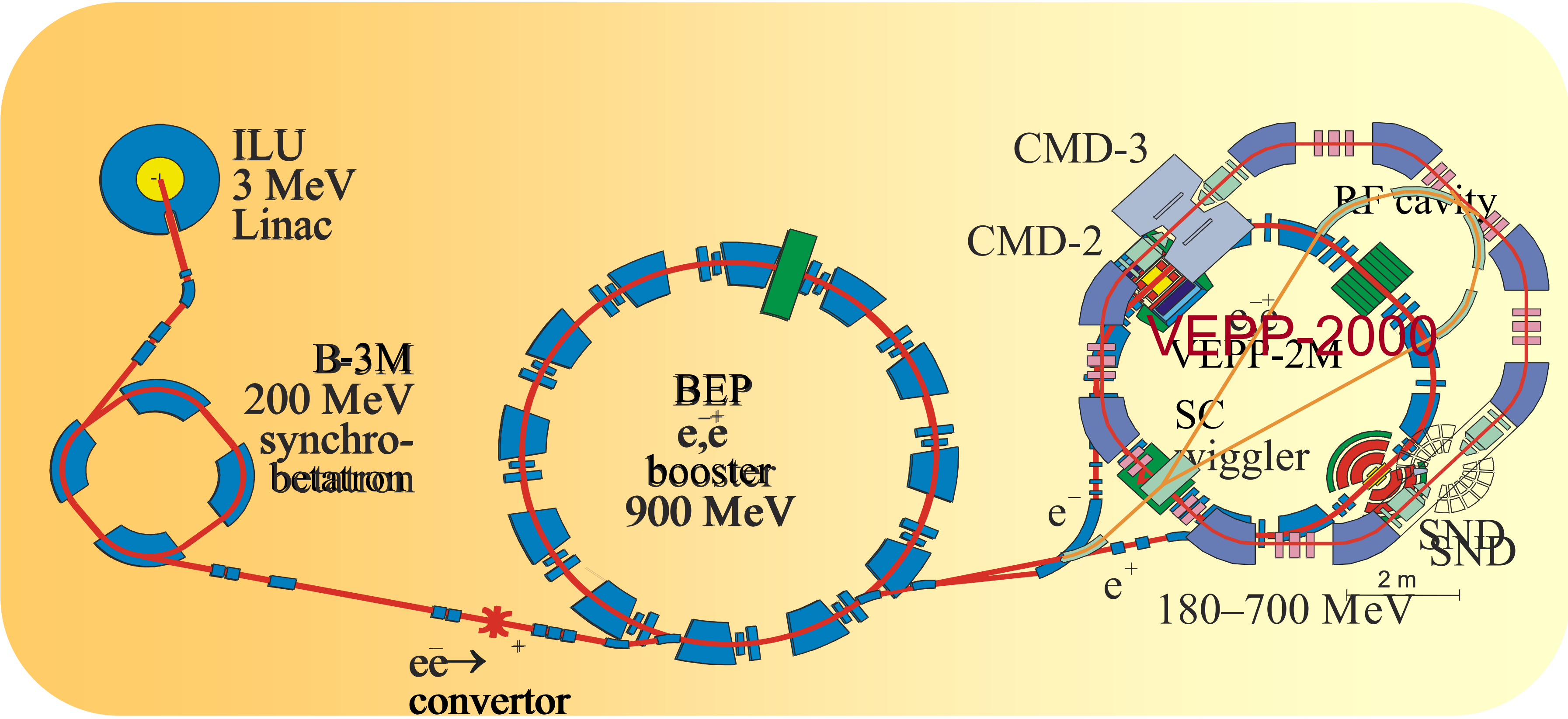
$$\frac{\Omega^{+}-\Omega^{-}}{\Omega} < 10^{-8}$$

ВЭПП-2М → ВЭПП-2000

ВЭПП-2М
1989-2000

2001-2007

ВЭПП-2000
2008

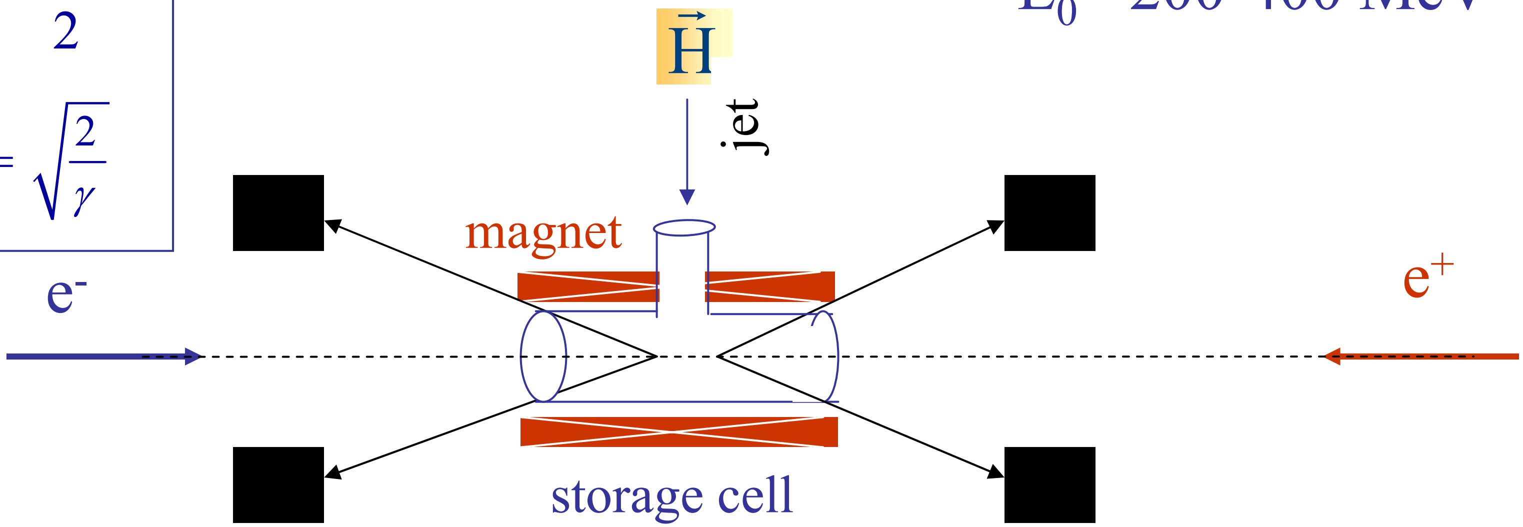


Moeller/Babba polarimeter

$$E = \frac{E_0}{2}$$

$$\theta = \sqrt{\frac{2}{\gamma}}$$

E_0 =200-400 MeV



$n=10^{13}$ - 10^{14} cm^{-2}

$$A_{\perp} = \pm\frac{1}{9}$$

$$A_{\parallel} = \pm\frac{8}{9}$$

$$\dot{N} = 10 - 100 \left(\frac{\text{Hz}}{\text{mA}\times\text{GeV}}\right)$$

Trap with ultra cold hydrogen T=0.5 K
$n=10^{16}$ cm^{-2}

P e r s p e c t i v e

(conclusion)

- VEPP-2000 (polarized beams transfer from buster BEP; τ_p =20 min E=900 MeV)
- E=200 MeV $V_{acc}\left(\varphi=\varphi_0\right)=100$ (V) $\frac{\Omega^+ - \Omega^-}{\Omega} \approx 10^{-10}$
- Decoherence time 200 sec.
- $\Delta t = 100$ sec. $\rightarrow 0.5\times10^9$ free anomalous spin precessions
- Moeler/Babba polarimeter ($\Delta t = 10^3$ sec $\rightarrow \Delta S/S \approx 5\%$)
- . $\frac{\gamma^- a^- - \gamma^+ a^+}{\bar{\gamma}} < 1\cdot10^{-10}$