Hadronic contribution to muon g-2

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"Quantum metrology and fundamental physical constants"
Sankt-Peterburg, December 2 - 4, 2008

Outline

- Introduction
- Theory
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- Light quarks(=hadrons) contribution
- Summary

1. Introduction: definition and motivation

Magnetic moment of spin-s particle

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{s}, \qquad \vec{s} = \vec{\sigma}/2, \qquad g - \text{Lande factor}$$

For Dirac particle (e.g. an electron)

$$(i\hat{D} - m)\psi = 0, \qquad D_\mu = \partial_\mu - ieA_\mu, \qquad \psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix}$$

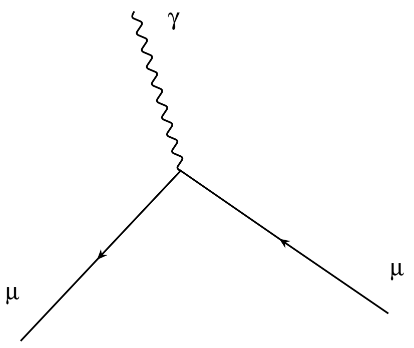
with ϕ a large component one finds

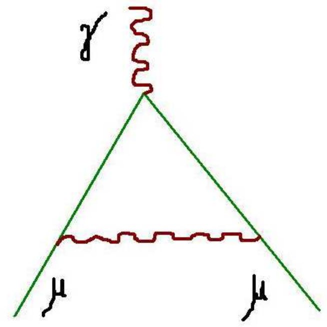
$$i\frac{d}{dt}\phi = \left[\frac{(\vec{p})^2}{2m} - \frac{e}{2m} \left(\vec{L} + 2\vec{S} \right) \cdot \vec{B} \right] \phi$$

B - magnetic field and $g = 2$. This is tree level result.

Experiment gave for the electron $g = 2(1 + a)$ with $a \neq 0$ - an anomaly.

It was realized that there are (quantum) corrections.





Schwinger result (1948)

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi}, \qquad \alpha = 1/137.04$$

explained the anomaly. One of the first calculations that made QED.

In SM the leptons are e, μ, τ and a_l are measured. For the electron a_e both experimental and theory results are precise. a_e dominates extraction of α

$$\alpha^{-1} = 137.03599911(46)[3.3 \text{ ppb}]$$

Heavy particles contribution to a_e scales as $(m_e/m_H)^2$.

Muon contribution: $m_e^2/m_\mu^2 \sim (1/200)^2 \sim \alpha^2$.

Hadronic contributions: $m_e^2/m_\pi^2 \sim (1/300)^2 \sim \alpha^2$.
Note the pion mass m_π that used instead of light quark mass m_q.

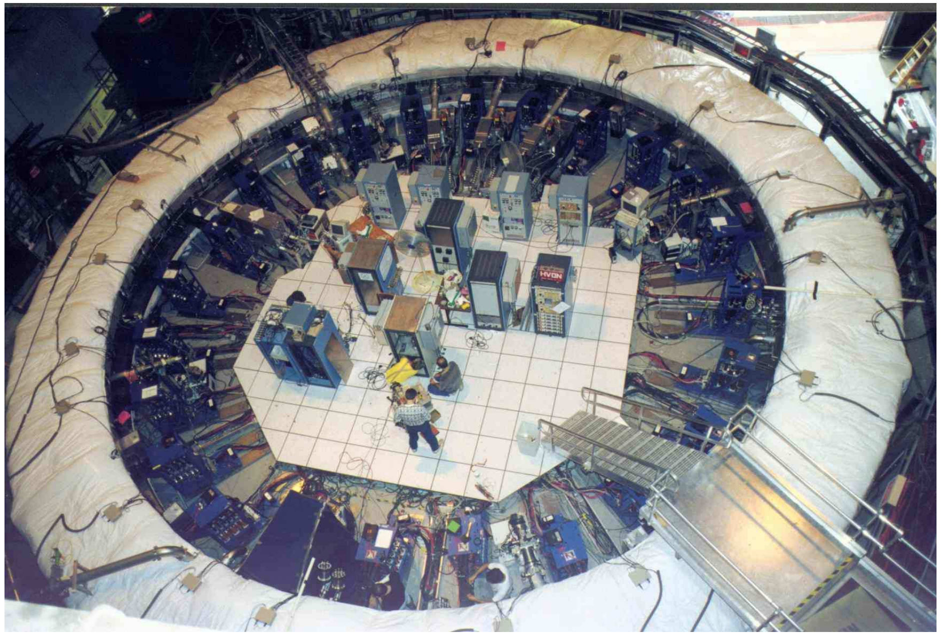
For the τ-lepton the sensitivity of a_τ to heavy particles with m_H is high, theory is reasonably accurate but no precise experimental data.

For the muon, both experiment and theory are pretty accurate and a_μ is the choice of present days precision analysis. Main goal: Search for new physics beyond SM.

Brookhaven experiment E821 result

$$a_\mu^{\rm exp} = 11659208.0(6.3) \times 10^{-10}$$

Precision: 0.54 ppm (Bennett *et al* (Muon (g-2) colaboration) 2004)



2. Theory of a_μ

Formally one considers the muon EM form factor

$$\langle \mu(p_1)|J^\mu|\mu(p_2)\rangle = \bar{u}(p_1)\left(\gamma^\mu F_E(q^2) + i\frac{\sigma^{\mu\nu}q_\nu}{2m}F_M(q^2)\right)u(p_2).$$

Then

$$a_\mu = F_M(0)$$

This definition allows for a straightforward computation of a_μ. In perturbation theory one uses Feynman diagrams to represent different contributions.

Classification of contributions in SM:
diagrams including only leptons - QED,
then diagrams with W, Z, H bosons - ElectroWeek (EW),
and diagrams with quarks - QCD

Theory: QED - direct PT calculation

General expression for leptonic contribution

$$a_\mu^{\mathrm{QED}} = A_1 + A_2(m_\mu/m_e) + A_2(m_\mu/m_\tau) + A_3(m_\mu/m_e, m_\mu/m_\tau)$$

A_1 – mass independent contribution (basically muon only),
$A_{2,3}$ – with electron and tau.

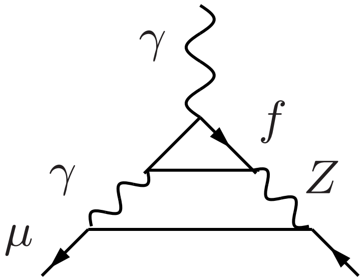
The five loop result (T.Kinoshita and collaborators during last decades)

$$a_\mu^{\mathrm{QED}} = (11\ 658\ 471.809 \pm 0.014|_{51} \pm 0.008|_\alpha \pm 0.004|_{\mathrm{m}}) \times 10^{-10}\,.$$

(taken from J.P.Miller, E. de Rafael, and B.L.Roberts (2007))

The uncertainty is small compared to $\sigma^{\exp} = 6.3 \times 10^{-10}$

Theory: EW - direct PT calculation



The EW corrections are well defined in the perturbation theory framework and have been computed with the two-loop accuracy. For $M_H = 150$ GeV

$$a_\mu^{\mathrm{EW}} = 15.4(0.2) \times 10^{-10}$$

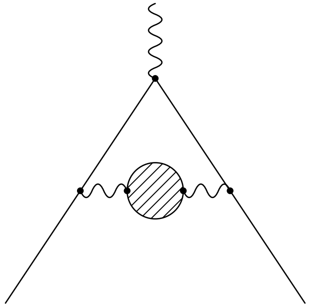
(from A.Czarnecki, W.J.Marciano, A.Vainshtein (2003)).
The contribution is suppressed by the masses of EW-particles and small (remind that $M_{\mathrm{Z,W}} \sim 100$ GeV).
Experimental uncertainty a_μ is 6.3×10^{-10}.

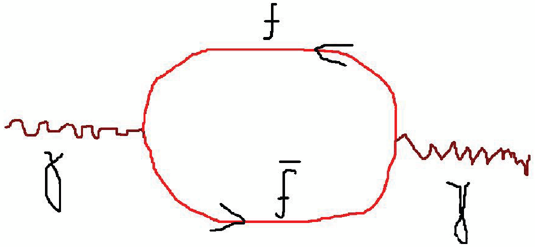
Theory: QCD – hadronic contribution

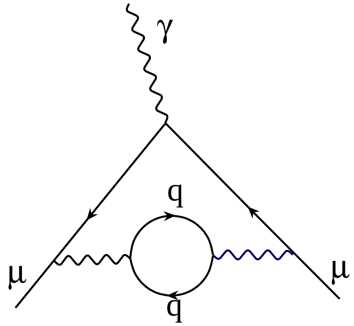
The hadronic contribution to MAMM is sensitive to IR region and cannot be computed in pQCD with light quarks (u, d, s). Massless approximation of QCD is good for describing high energy inclusive processes (pQCD for OPE) and fails for a_μ where IR region is crucial.

The current masses of light quarks are too small to provide a proper IR cutoff, i.e. $m_q \ll \Lambda_{\rm QCD}$, $m_{u,d} \sim 5$ MeV, $m_s \sim 150$ MeV and $\Lambda_{\rm QCD} \sim 0.5 - 1$ GeV. Explicit models of confinement are required for the quantitative analysis (i.e. compute in terms of massive hadrons).

This constitutes a main difficulty of the theoretical analysis in SM.
Also no attempts of the calculation on the lattice.







Indeed, for $m_q \ll m_\mu = 106 \text{ MeV}$

$$a_\mu^{\text{vac(2)q}} = \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{1}{3} \ln \frac{m_\mu}{m_q} - \frac{25}{36}\right)$$

is singular for $m_q \to 0$ while $m_{u,d} \sim 5 \text{ MeV}$

Writing $a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm had}$ one finds

$$a_\mu^{\rm had}|_{\rm th} = (720.8 + 6.3|_{\rm exp} + 0.2|_{\rm EW}) \times 10^{-10}$$

Accuracy is 1%. One has to explain this number within QCD beyond PT. In general terms the hadronic contribution to MAMM is determined by the correlation functions of EM hadronic currents.
At LO (α^2) only two-point correlation function emerges

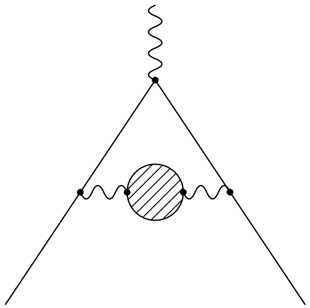
$$\Pi_2(x) \sim \langle T j_\mu^{had}(x) j_\nu^{had}(0)\rangle \, .$$

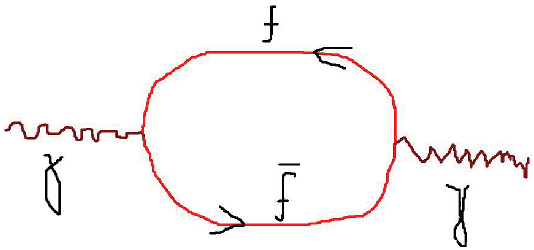
At NLO (α^3) the four-point correlation function appears

$$\Pi_4(x,y,z) \sim \langle T j_\mu^{had}(x) j_\nu^{had}(y) j_\alpha^{had}(z) j_\beta^{had}(0)\rangle \, .$$

As the source for the EM current is readily available for a wide range of energies one tries to extract these functions from experiment.

Hadronic contribution at the leading order in α





At LO the hadronic contribution is described by the correlator

$$i\int \langle T j_\mu^{had}(x) j_\nu^{had}(0)\rangle e^{iqx} dx = (q_\mu q_\nu - g_{\mu\nu} q^2)\Pi^{\rm had}(q^2)$$

The correlator is transverse that gives a single function $\Pi^{\rm had}(q^2)$. Analytic properties are known.

$\Pi^{\text{had}}(q^2)$ gives a contribution to MAMM

$$a_\mu^{\text{had}}(\text{LO}) = 4\pi \left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) \text{Im}\ \Pi^{\text{had}}(s)$$

with the one-loop kernel of the form

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\mu^2}}, \qquad K(0) = \frac{1}{2}, \qquad K(s)|_{s\to\infty} = \frac{m_\mu^2}{3s}.$$

Here $\text{Im}\ \Pi^{\text{had}}(s) = \text{Im}\ \{\Pi^{\text{had}}(q^2)|_{q^2=s+i0}\}$

The problem is that the integral is sensitive to small s and cannot be computed in PT for light quarks (u, d, s).

However for heavy c and b quarks one can use QCD perturbation theory instead of data on J/ψ ($\bar{c}c$) and $\bar{b}b$ meson families). For the c quark with the mass $m_c = 1.6$ GeV

$$a_\mu(\mathrm{LO; c}) = 6.9 \times 10^{-10} \,.$$

The b-quark contribution for $m_b = 4.8$ GeV reads

$$a_\mu(\mathrm{LO; b}) = 0.2 \times 10^{-10} \,.$$

Main uncertainty is due to quark masses but it is small.

For light quarks at the leading order in α the function $\mathrm{Im}\ \Pi^{\mathrm{had}}(s)$ can be extracted from e^+e^- annihilation data

Experimental $R^{\rm exp}(s)$ ratio

$$R^{\rm exp}(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}\,, \qquad s = (p_{e^+} + p_{e^-})^2$$

is identified with $R^{\rm th}|_{LO}(s)$ taken at the leading order in α as

$$R^{\rm th}|_{LO}(s) = 12\pi \text{Im}\ \Pi^{\rm had}(s)$$

Then

$$a_\mu^{\rm had}(\text{LO}) = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{R^{\rm exp}(s) K(s)}{s} ds\,.$$

The contributions to MAMM based on data are well studied:

$$a_\mu^{\rm had}({\rm LO}) = 696(7)\times 10^{-10} \qquad \text{(Davier } et\ al \text{ (2003));}$$
$$a_\mu^{\rm had}({\rm LO}) = 695(9)\times 10^{-10} \qquad \text{(Jegerlehner (2004));}$$
$$a_\mu^{\rm had}({\rm LO}) = 694(5)\times 10^{-10} \qquad \text{(Yndurain (2004));}$$
$$a_\mu^{\rm had}({\rm LO}) = 692(7)\times 10^{-10} \qquad \text{(Hagiwara } et\ al \text{ (2004));}$$

$$a_\mu^{\rm had}({\rm LO}) = 691(4)\times 10^{-10} \qquad \text{(Davier (2006));}$$
$$a_\mu^{\rm had}({\rm LO}) = 689(5)\times 10^{-10} \qquad \text{(Hagiwara } et\ al \text{ (2006))}$$

The error is 1% and of the order of experimental uncertainty for a_μ

NLO contribution

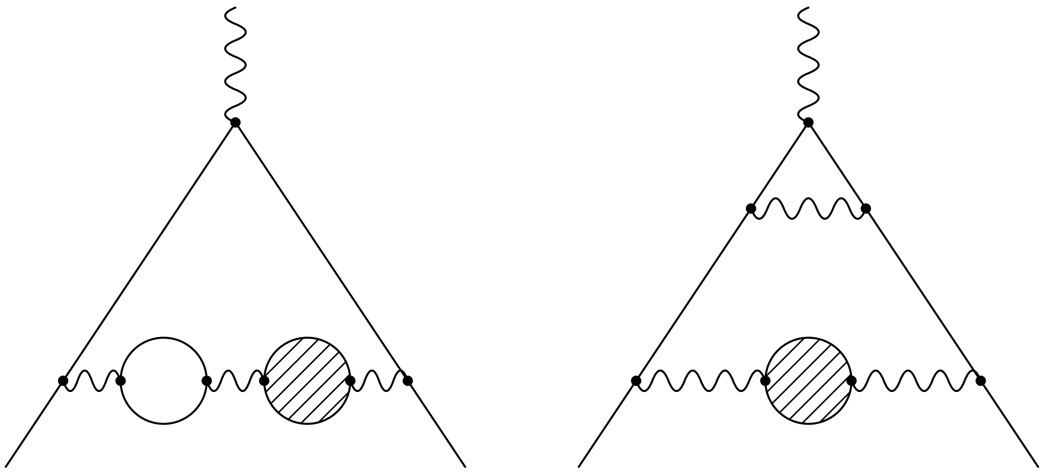
In NLO there is less transparency with determining hadronic contributions.

Basically there are two new features:
Experiment – interpretation of data in the NLO calculations is more involved. The problem is to avoid double counting as a part of contributions has been accounted through the data at LO.

Theory – a new correlation function Π_4 which is much more complicated than Π_2 enters the game. At present there is no experimental determination of Π_4. One has to rely on models for this function. It is difficult to control the accuracy of such models that introduces model dependence in the calculation of the NLO and makes predictions less definite than in LO.

Two-point function Π_2-related contributions at NLO:

straightforward computation. Two types



a) b)

Type a) is related to vacuum polarization and calculable technically.
The problem is a mixture with LO data of e^+e^--annihilation.

Type b) gives a new contribution – vertex related corrections.

Vertex related NLO contribution is an integral of $\mathrm{Im}\ \Pi^{\mathrm{had}}(s)$ with the two-loop kernel $K^{(2)}(s)$

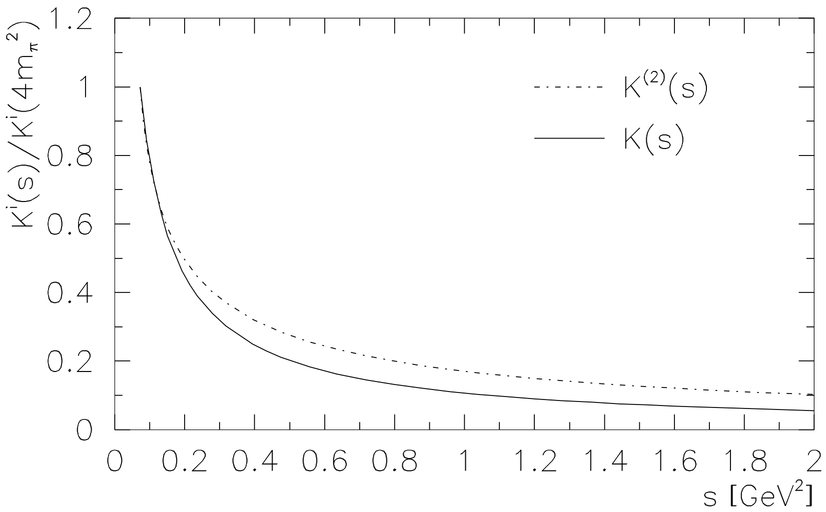
$$a_\mu^{\mathrm{had}}(\mathrm{NLO}) = 4\pi \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty \frac{ds}{s} K^{(2)}(s) \mathrm{Im}\ \Pi^{\mathrm{had}}(s).$$

The expression for the kernel $K^{(2)}(s)$ is known (Barbieri,Remiddi (1975)). It has an expansion at small m_μ^2/s (independently by Krause (1997))

$$K_{\mathrm{ver}}^{(2)}(s) = 2\frac{m^2}{s}\left(\frac{223}{54} - \frac{\pi^2}{3} - \frac{23}{36}\ln\left(\frac{s}{m^2}\right)\right)$$

$$+\left(\frac{m^2}{s}\right)^2\left(\frac{8785}{1152} - \frac{37\pi^2}{48} - \frac{367}{216}\ln\left(\frac{s}{m^2}\right) + \frac{19}{144}\ln^2\left(\frac{s}{m^2}\right)\right)$$

The form of $K^{(2)}(s)$ is close to that of $K(s)$
(fig from S.Groote,J.Korner,AAP (2002))

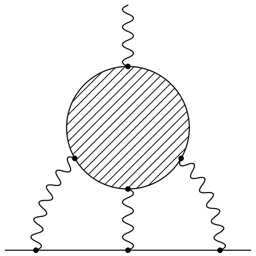


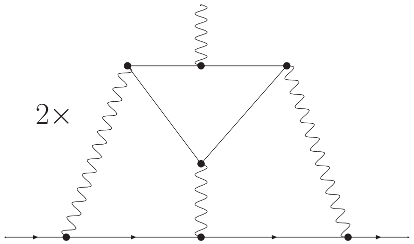
Total result for NLO related to Π_2 (2007):

$$a_\mu^{\rm had}(\rm NLO) = -9.8(0.1) \times 10^{-10}$$

Sign is fixed: $K^{(2)}(s)$ is negative

Four-point function Π_4-related contributions: Light-by-Light type





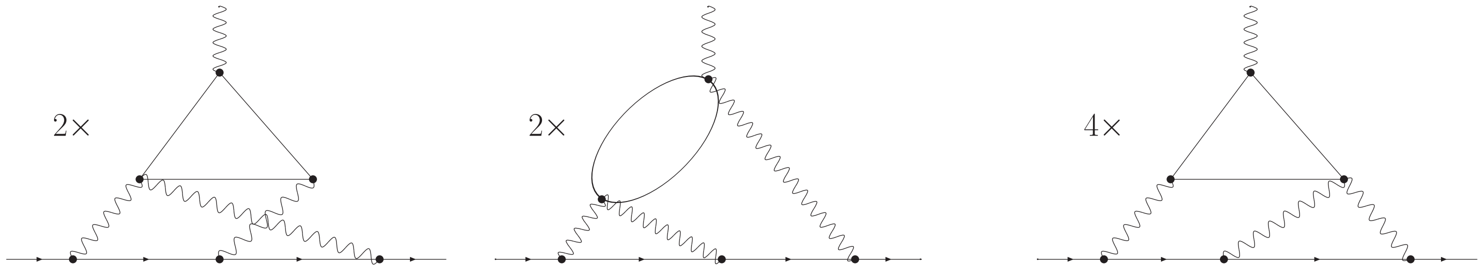
Contribution of heavy quarks (c,b) known analytically (S.Laporta, E.Remiddi (1993))

$$a_\mu^{\rm had}({\rm LbL; heavy}) = 0.2 \times 10^{-10}$$

for c quark with $m_c = (1.6 \pm 0.2)$ GeV and negligible for b quark with $m_b = (4.8 \pm 0.2)$ GeV.

Contribution of light quarks – problems since no PT QCD.

Calculation using lightest hadrons: charged π, neutral π, kaons,... Charged pions (kaons,...): Scalar QED



To quantitatively handle contributions from the four-point function a quantum field model for pions given by the Lagrangian

$$L_{\text{low energy}} = |D_\mu \pi|^2 - m_\pi^2 \pi^2, \quad D_\mu = \partial_\mu - ieA_\mu$$

is used. Computed numerically by T.Kinoshita and collaborators (1985,...)

Analytical results in scalar QED for charged pions
(J.H.Kuhn,A.I.Onishchenko,AAP,O.L.Veretin (2003))

$$
\begin{aligned}
a_\mu(\gamma\gamma;\mathrm{sQED}) &= \frac{m^2}{M^2}\left(\frac{1}{4}\zeta_3 - \frac{37}{96}\right) \\
&+\frac{m^4}{M^4}\left(\frac{1}{8}\zeta_3 + \frac{67}{6480}\zeta_2 - \frac{282319}{1944000} + \frac{67}{12960}L^2 + \frac{7553}{388800}L\right) \\
&+\frac{m^6}{M^6}\left(\frac{19}{216}\zeta_3 + \frac{157}{36288}\zeta_2 - \frac{767572853}{7112448000} + \frac{1943}{725760}L^2 + \frac{51103}{7620480}L\right) \\
&+\frac{m^8}{M^8}\left(\frac{11}{160}\zeta_3 + \frac{943}{432000}\zeta_2 - \frac{3172827071}{37507050000} + \frac{8957}{6048000}L^2 + \frac{22434967}{7620480000}L\right)
\end{aligned}
$$

for $m = m_\mu$ and $M = m_\pi$

Compare to fermionic QED (a check of S.Laporta, E.Remiddi (1993))

$$a_\mu(\gamma\gamma;\mathrm{QED}) = \frac{m^2}{M^2}\left(\frac{3}{2}\zeta_3 - \frac{19}{16}\right)$$

$$+\frac{m^4}{M^4}\left(\frac{13}{18}\zeta_3 - \frac{161}{1620}\zeta_2 - \frac{831931}{972000} - \frac{161}{3240}L^2 - \frac{16189}{97200}L\right)$$

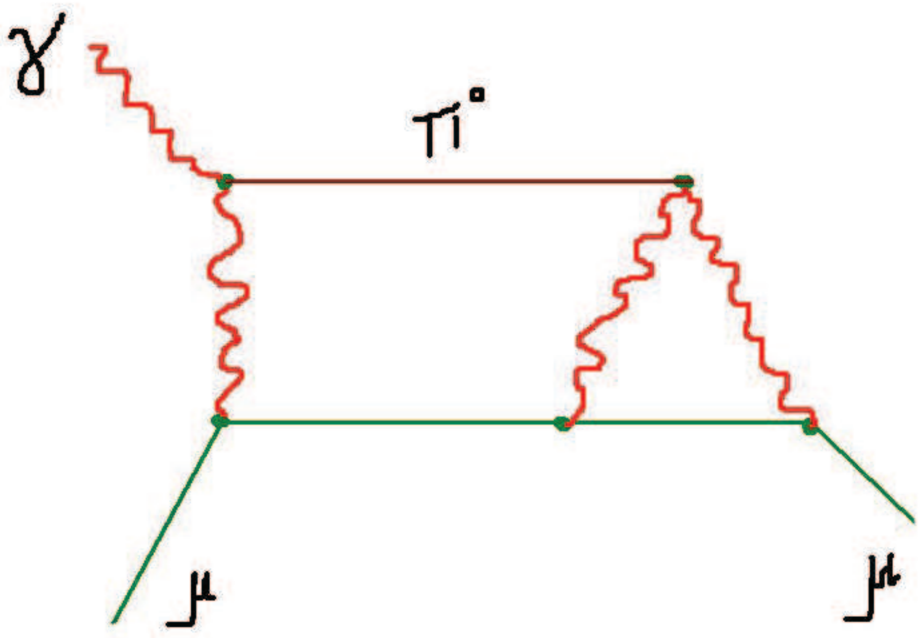
Scalar QED

$$a_\mu(\gamma\gamma;\mathrm{sQED}) = \frac{m^2}{M^2}\left(\frac{1}{4}\zeta_3 - \frac{37}{96}\right)$$

$$+\frac{m^4}{M^4}\left(\frac{1}{8}\zeta_3 + \frac{67}{6480}\zeta_2 - \frac{282319}{1944000} + \frac{67}{12960}L^2 + \frac{7553}{388800}L\right)$$

where $L = \ln(M^2/m^2)$, m and M denoting muon and heavy particle mass respectively, $\zeta_2 = \zeta(2) = \pi^2/6$, $\zeta_3 = \zeta(3)$.

Light-by-Light type – Neutral pion contribution: the four-point function Π_4 reduces to the contribution of the lighest possible hadron π^0:



Problems: Local vertex (anomaly) gives UV divergence, form factors are unknown...

LBL-type: model dependent results for light quarks

$$a_\mu^{\rm had}({\rm lbl}) = 8.3(3.2) \times 10^{-10} \qquad ({\rm Bijnens,\ Prades\ (2002)})$$

$$a_\mu^{\rm had}({\rm lbl}) = 8.9(1.7) \times 10^{-10} \qquad ({\rm Kinoshita\ (1998),\ sign!})$$

$$a_\mu^{\rm had}({\rm lbl}) = 10.6(1.0) \times 10^{-10} \qquad ({\rm Dorokhov\ (2005)})$$

$$a_\mu^{\rm had}({\rm lbl}) = 13.6(2.5) \times 10^{-10} \qquad ({\rm Melnikov,\ Vainshtein\ (2004)})$$

$$a_\mu^{\rm had}({\rm lbl}) = 14.3(1.6) \times 10^{-10} \qquad ({\rm Pivovarov\ (2001)})$$

Total theory (in units 10^{-10})

$$a_\mu^{\rm had}|_{\rm SM} = 690 \pm 6 + (-9.8 \pm 0.1) + (11 \pm 4) = 691 \pm 6 \pm 4$$

Remind the experiment

$$a_\mu^{\rm had}|_{\rm exp} = (720.8 + 6.3|_{\rm exp} + 0.2|_{\rm EW}) \times 10^{-10}$$

There is a $3.4\ \sigma$ discrepancy
(as from J.P.Miller, E. de Rafael, and B.L.Roberts (2007)).

Calculation based on interpolation of data in the effective theory

Bogolubov compensation scheme for light quarks: effective theory reduction $SU(3)_c \otimes U(1)_{\rm EM} \rightarrow U(1)_{\rm EM}(m^*)$ or (schematically at the level of Lagrangians)

$$L_{\rm QCD} \rightarrow \bar{\psi}(i\hat{D}_{\rm EM} - m^*)\psi$$

Clearly, high energy asymptotics of correlation functions Π_2 and Π_4 are correct while IR cutoff by given by m^*.

Important point is reformulation of the calculation in Euclidean domain and for $m^* > m_\mu$ one can use m_μ/m^* expansion.

Assumption: m^* extracted from Π_2 can be used for Π_4 for calculation of MAMM

Euclidean domain representation at LO

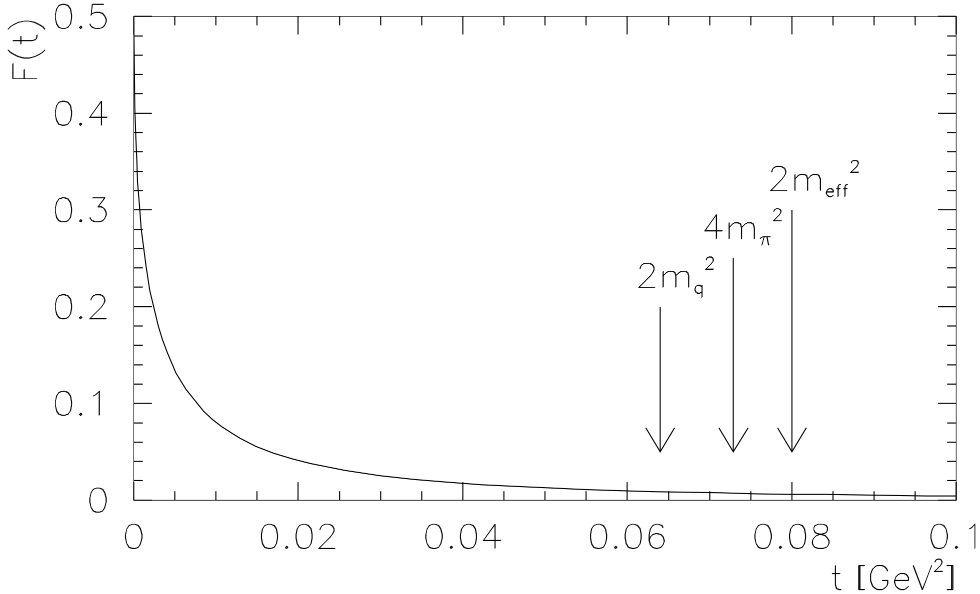
$$\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds}{s}K(s)\mathrm{Im}\ \Pi^{\mathrm{had}}(s)=\int_0^{\infty}\left(-\frac{d\Pi^{\mathrm{had}}(-t)}{dt}\right)F(t)dt,$$

with (S.Groote,J.Korner,AAP (2002))

$$F(t)=\frac{1}{2}\left(\frac{t+2m^2-\sqrt{t^2+4m^2t}}{t+2m^2+\sqrt{t^2+4m^2t}}\right)=\frac{2m^4}{\left(t+2m^2+\sqrt{t^2+4m^2t}\right)^2}.$$

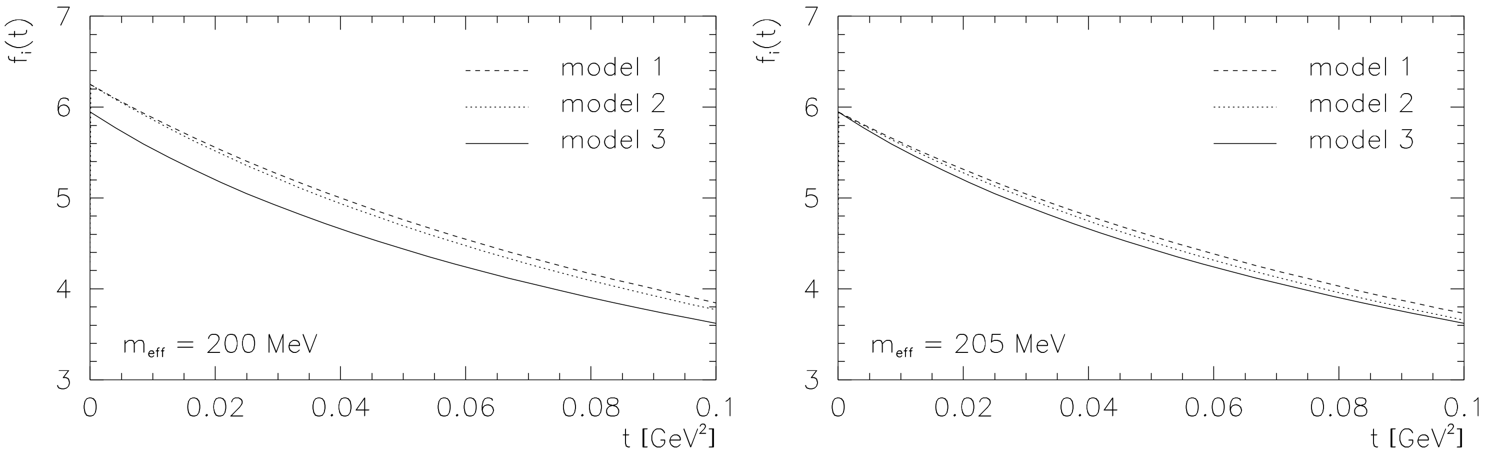
$-\Pi^{\mathrm{had}}(-t)/dt$ is Adler function in Euclidean domain. Can be computed in PT far from the origin. $F(t)$ is a weight function.

$F(t)$ dies off fast at large t but $F(0) = 1/2$. Similar to $\delta(t)$



The integral is IR sensitive and can be computed using m^* as IR cutoff.

Model spectrum is obtained from LO analysis



that gives $m^* = 2m_{\rm eff}/\sqrt{5} = 183\ {\rm MeV}$. This values reproduces accurately NLO results for a_μ related to Π_2. Used for Π_4 it allows for the estimate of LbL conribution in the form

$$a_\mu^{\rm had}({\rm lbl}) = 14.3(1.6) \times 10^{-10} \qquad ({\rm Pivovarov\ (2001)})$$

4. Summary

MAMM is important observable in SM precisely known both theoretically and experimentally.

Hadronic contribution to a_μ is a main problem in theory

The error 1% (the level of experimental error) is difficult to handle for hadrons, disentangling LO and NLO contributions in precision data at low energy. Need for better experimental data.

The discrepancy between theory and experiment at about 3σ (maybe less as new data from Novosibirsk presented at the Workshop suggest).

Validity of SM is not seriously challenged and existence of New Physics is still an open question