ELECTRIC DIPOLE MOMENTS, FROM ELECTRON TO t-QUARK

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1. Introduction

	e	n	p	μ
d/e, cm	$(0.7 \pm 0.7) \times 10^{-27}$ atomic spectroscopy, Tl (Berkeley)	$(-0.1 \pm 0.36) \times 10^{-25}$ ultra-cold neutrons (ILL)	$< 0.5 \times 10^{-23}$ atomic spectroscopy, Hg (Seattle, Novosibirsk)	$(0.3 \pm 0.3) \times 10^{-18}$ muon storage ring (Brookhaven)

Table 1. Best present limits on dipole moments of elementary particles

2. τ-lepton EDM contribution to electron dipole moment

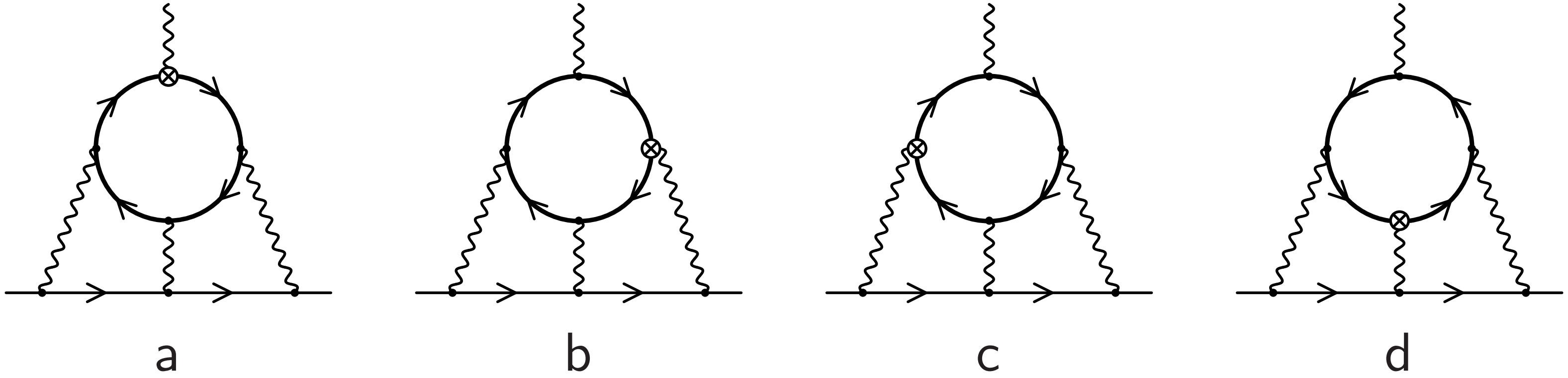

Figure 1

Loops are formed by τ lines, and lower solid lines are electron ones. Upper wavy lines correspond to external electric field. Crossed vertices refer to electromagnetic interaction of τ EDM. All six permutations of electromagnetic vertices on electron line should be considered. Contributions of diagrams 2b and 2c are equal.

General structure of resulting contribution to electron EDM is rather obvious:

$$\Delta d_e = a \frac{m_e}{m_\tau} \left(\frac{\alpha}{\pi} \right)^3 d_\tau \,, \tag{1}$$

where a is a numerical factor (hopefully, on the order of unity). Factor m_e originates from necessary helicity-flip on electron line; then $1/m_\tau$ is dictated by dimensional arguments.

Contribution of diagram 1a to a is

$$a_1 = \frac{3}{2}\zeta(3) - \frac{19}{12}; \tag{2}$$

here and below ζ is Riemann ζ-function. a_1 is numerically small. Contribution of Figs. 1b–d to a is

$$a_2 = \frac{9}{4}\zeta(3) - 1\,. \tag{3}$$

Final result for numerical coefficient is

$$a = a_1 + a_2 = \frac{15}{4}\zeta(3) - \frac{31}{12} = 1.924\,. \tag{4}$$

Combining this result with the experimental one for the electron EDM, we arrive at

$$d_\tau/e = (1 \pm 1) \times 10^{-16}\,\mathrm{cm}\,. \tag{5}$$

In fact, results (2) and (3) refer to somewhat different ranges of entering momenta. For (3) all three momenta are hard, on the order of magnitude about m_τ, but for (2) only two of them belong to this region, and the third one, that of the outer photon, is soft, of vanishing momentum. Still, one may expect that effective EDM interaction is formed at momenta much higher than m_τ, so that this difference is not of much importance. Besides, contribution of diagram 1a is anyway numerically small. Thus, result (2) is valid at least for all momenta about $m_\tau \sim 1$ – 2 GeV.

Our result (5) looks rather modest as compared to those derived from the accelerator experiments (discussed below). However, all those accelerator data refer to quite different kinematical region: therein invariant momentum transfer to photon $\sqrt{q^2}$ changes from 10 to 200 GeV.

3. Dipole moments of heavy quarks and electron EDM

Information on dipole moments d_Q of heavy quarks can be obtained from diagrams analogous to those presented in Figs. 1, but with quarks in fermion loops, instead of τ-lepton. Because of strong interactions, these contributions to the electron EDM cannot be calculated accurately. Still, reasonable estimate for them, following from the analogy between the two problems, looks as follows (see (1)):

$$\Delta d_e = a_Q \, 3 \, \nu^3 \, \frac{m_e}{m_Q} \left(\frac{\alpha}{\pi} \right)^3 d_Q \, ; \tag{6}$$

here m_Q is the quark mass, ν is the quark charge in the units of e ($\nu_b = -1/3$, $\nu_{c,t} = 2/3$). In estimates we put overall numerical factor $a_Q \simeq 1$. Estimates are straightforward and result in upper limits on d_Q/e about $10^{-14} - 10^{-16}$ cm. More detailed values of them will be presented somewhat later in Table 2.

4. Dipole moments of heavy quarks and neutron EDM

Dipole moment of heavy quark Q generates EDM of light quark q via diagram (Cordero-Cid et al.)

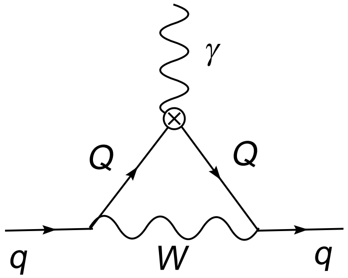

Figure 2

Corresponding contributions to light quark EDMs are as follows:

$$\Delta d_q = \frac{\alpha}{16\pi \sin^2\theta_w} |V_{Qq}|^2 \frac{m_q m_Q}{m_w^2} \left(\ln \frac{\Lambda^2}{m_w^2} - 2 \right) d_Q, \tag{7}$$

if $Q = c, q = d$ or $Q = b, q = u$; and

$$\Delta d_d = \frac{\alpha}{16\pi \sin^2\theta_w} |V_{td}|^2 \frac{m_d}{m_t} \left(\ln \frac{\Lambda^2}{m_t^2} - 4 \right) d_t, \tag{8}$$

if $Q = t, q = d$. Here $\sin^2\theta_w = 0.23$, and V_{Qq} are coefficients of Kobayashi-Maskawa matrix.

Relations (7) and (8) have different structures since $m_{c,b} \ll m_w$, while $m_t > m_w$.

Contribution of $k_\mu k_\nu / m_w^2$ is logarithmically divergent (which is missed by Cordero-Cid et al.) In our estimates we assume that both $\ln(\Lambda^2/m_w^2) - 2$ and $\ln(\Lambda^2/m_w^2) - 4$ are on the order of unity.

As to light quark EDMs, we assume that they are on the same order of magnitude as neutron EDM:

$$d_{u,d} \sim d_n.$$

Of course, both these assumptions, combined with neglect of strong interaction of quarks, make these our estimates less definite than those based on electron EDM. Results of these our estimates are presented in Table 2.

	c	b	t
m_Q, GeV	1.25	4.5	175
d_Q/e, cm derived with d_e	$\lesssim 3 \times 10^{-16}$	$\lesssim 7 \times 10^{-15}$	$\lesssim 4 \times 10^{-14}$
d_Q/e, cm derived with d_n	$\lesssim 10^{-15}$	$\lesssim 3 \times 10^{-15}$	$\lesssim 3 \times 10^{-15}$

Table 2. Limits on quark dipole moments from electron and neutron EDMs

It is only natural that for most heavy t-quark bound from d_n dominates.
For c- and b-quarks, both approaches result in comparable upper limits.

5. Upper limits on dipole moments from e^+e^- annihilation

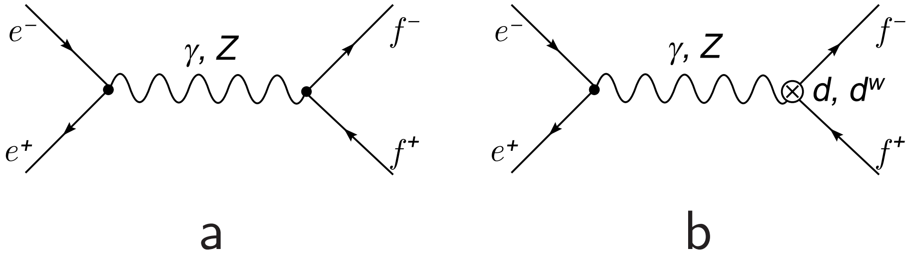

Figure 1: a) Regular electromagnetic amplitude; b) EDM contribution

Here final fermions f are τ-leptons or heavy quarks, c and b. Regular amplitude is due to intermediate γ and Z-boson with usual electromagnetic and neutral-current couplings, and fermions are produced in triplet states, $^3S_1$ and $^3D_1$ for vector couplings, and $^3P_1$ for axial one. However, when produced via CP-odd EDM vertex, fermions are in singlet $^1P_1$ state.

Triplet and singlet amplitudes do not interfere in cross-sections for unpolarized f!

They do interfere however if spin correlations of final particles are measured.

On the other hand, the EDM amplitude, being squared, contributes to the total annihilation cross-section $e^+e^- \longrightarrow \tau^+\tau^-$. Moreover, this ${}^1P_1$ contribution to the differential cross-section behaves as $\sin^2\theta$, as distinct from the regular $1+\cos^2\theta$ (in the ultrarelativistic limit) for the triplet channels.

	d_τ/e, cm	$d_c/e, cm$	d_b/e, cm
del Aguila, Sher	$< 1.4 \times 10^{-16}$		
Inami (Belle)	$< 4.5 \times 10^{-17}$		
Escribano, Masso	$< 10^{-17}$	$< 0.9 \times 10^{-16}$	$< 10^{-16}$
Blinov, Rudenko	3×10^{-17}	5×10^{-17}	2×10^{-17}

Table 3. Limits on dipole moments from e^+e^- annihilation

Our results for d_τ, d_c, and d_b, derived from upper limits on electron and neutron dipole moments, look modest as compared to those presented in Table 3. However, all accelerator data refer to kinematical regions where invariant momentum transfer to photon $\sqrt{q^2}$ belongs to the interval from 10 to 200 GeV. Meanwhile, both for d_τ and d_c typical values of $\sqrt{q^2}$ are $\simeq 1$ GeV. As to our result for d_b derived from d_n, therein $\sqrt{q^2} \longrightarrow 0$ at all.

Thus, here we deal not with rivalry between, but with

complimentarity of low-energy and high-energy results.